

2016年12月17日

兵庫教育大学長 殿

受入研究者

所属・職 理数系教育コース・教授

氏名 小池敏司



外国人研究者短期招へいプログラム研究成果報告書

外国人招へい研究者の研究経過について、下記のとおり報告します。

1. 外国人招へい研究者 氏名 (所属・職・国籍) Karim Bekka (レンヌ大学・准教授・フランス)
2. 研究課題名 滑らかな写像の相対有限既定性に関する研究 (英訳名) Research on relative finite determinacy of differentiable maps
3. 期間 2016年10月16日 ~ 2016年12月15日 (61日間)
4. 主な研究協力者 氏名 (所属・職・氏名) 埼玉大学大学院理工学研究科・教授・福井敏純

(注) 必ず招へい研究者の作成した Research Report と併せて、招へい期間終了後1か月以内に、提出してください。 (裏面につづく)

## 5. 滞在中の日程

年月日	訪問先名称・訪問内容（研究討議・講演・視察等）
平成 28 年 10 月 16 日	関西空港到着、兵庫教育大学に移動。12 月 5 日まで受入研究者と共同研究を行う。その間、10 月 26 日と 11 月 2 日の 2 回、学内講会で講演する。
平成 28 年 12 月 6 日	京都大学数理解析研究所に移動、12 月 9 日まで特異点論研究集会に参加。12 月 8 日に、受入研究者との共同研究成果について発表。
平成 28 年 12 月 9 日	兵庫教育大学に移動。12 月 14 日まで受入研究者と共同研究を行う。
平成 28 年 12 月 15 日	関西空港より帰国。

(注) 来日日及び離日を含めて記入してください。

## 6. 研究討議・研究協力等実施の状況とその成果

構造安定性問題に関連して導入された、一点でのジェットの十分性、写像の有限既定性の概念に対して、多くの判定法・特徴付けが与えられている。この共同研究では、一点を含む任意の閉集合における相対ジェットの場合に一般化した十分性や写像の有限既定性について、判定法・特徴付けを与えた。本研究は数年前から続けられて来ており、今回の研究で 40 頁を超える共著論文として、完成に近付いた。今後、欧州の数学雑誌に投稿を予定している。

## 7. 外国人招へい研究者事業に対する意見・要望等

この事業そのものに対しては、特にありません。

別記第4号様式【招へい研究者作成/By Fellow】

\* Please submit your research report to HUTE through your host researcher within one month after the end of your Fellowship Period in Japan.

To President, HUTE

**HUTE SHORT-TERM FELLOWSHIP PROGRAM**  
**FOR RESEARCH IN JAPAN**  
**RESEARCH REPORT**

(Cover Page)

Affiliation: Université de Rennes 1

Name of the Host Researcher: BEKKA Karim

Fellowship Period: From 15 October 2016 To 15 December 2016

Title of the Research:

Geometry and Topology of Singularities

Date: 16 December 2016

Your Signature: 

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\* Future Contact Information

(If you wish HUTE to maintain contact with you after the completion of your fellowship)

(Office/Home)

Fax Number

E-mail Address

Address :

(Rennes)

## NOTES FOR WRITING THE RESEARCH REPORT

Please prepare your Research Report in English or Japanese written within three to four pages in length.

1. Items to be included:
  - (a) Outline of academic activities Discussions, lecture tour, etc.
  - (b) Impressions and thoughts on the present state of science in Japan in your field
  - (c) Comments or suggestions to HUTE, especially concerning this program
  - (d) Other comments
2. If available, please attach photographs of yourself taken during your academic activities, such as experiments, discussions or research trips in Japan.
3. You are required to submit this Research Report within one month of the end of your fellowship period in Japan through your host researcher.

(和訳)

研究報告書を書くための注意事項

長さ 3～4 ページ以内で英語か日本語で研究報告書を作成してください。

1. 含めるべき項目:
  - (a) 研究活動, 研究討議, 講演旅行等の概要
  - (b) あなたの専門分野における日本の科学の現状に関する印象と考え
  - (c) 特にこのプログラムに関する兵庫教育大学への意見もしくは提案
  - (d) その他の意見等
2. できるならば, 日本での実験, 討議または研究旅行等のような研究活動の間に撮られた自分の写真を添付してください。
3. 研究終了後 1 カ月以内に受入研究者を通して, 研究報告書を提出してください。

# Research Report

## Purpose of the visit

- Visiting professor **Satoshi KOIKE** at Hyogo University of Teachers Education.
- Participation to : Symposium on Singularity theory of differential maps at RIMS (Research Institut for Mathematical Sciences) Kyoto University. From December 6th, 2016 to December 9th, 2016

## Description of the work carried out during the visit:

### A) Symposium on Singularity theory of differential maps at RIMS (Research Institut for Mathematical Sciences) Kyoto University.

At this conference, I gave a talk and had the opportunity to exchange and discuss mathematics with some of the expert in singularity theory. My talk held on the 8<sup>th</sup> of December 2016.

The purpose of this talk is to give a presentation of the new ideas in the theory of efficiency and determinacy for non-isolated singularity, I have developed with Professor Satoshi Koike during the last two years. More details will be given below in the reaserch paragraph.

### B) Colloquium talks to graduate students:

I gave two lectures in the colloquium of the University: the first lecture held on Wednesday 26 October and the second one on Wednesday 2 November.

#### 1) The title of the first lecture: " Polynomial equations: real solutions "

##### Abstract:

One of the fundamental problems in mathematics and its applications is that of solving polynomial equations. The Polynomial equations arise in many mathematical models in science and engineering. In such applications one is typically interested in solutions over the real numbers instead of the complex numbers.

This study of real roots of polynomial systems is considerably more difficult than the study of complex roots. While the number of complex roots of a univariate polynomial is typically equal to its degree ( fundamental theorem of algebra), the number of real roots is severely constrained by the number of terms and the signs of its coefficients

In this talk, I will present some methods that were developed to study the real roots of univariate polynomials (Descartes' rule, Budan-Fourier,...), which are now standard tools in some applications of mathematics.

In the second part of the talk, I will discuss how in many variables the systems that arise naturally typically possess some geometric or combinatorial structure that may be exploited to study their solutions (Bezout, Berstein, Khovanskii, ...).

#### 2) The title of the second lecture: " Introduction to singularity theory and it's applications "

**Abstract:**

The study of singularities is a central topic in almost all areas of mathematics.

A singularity refers to a point where the nice structure breaks down.

for example, for a geometric space, a singularity means a point at where the space fails to have a manifold structure (smooth structure). The total classification of singularities is a difficult question.

The singularity theory gives a rigorous body of mathematics which enables us to study phenomena which re-occur in many situations.

In this talk, I will start by introducing the notion of singularity for functions and curves, and the algebraic and analytic tools to study it.

Then I'll then discuss some of its applications like in robotics, computer vision, like recovery of shape from profiles (outlines, apparent contours), the singularities here are those which arise from projection of a surface into a view plane.

**C) Research**

During my stay, we have extended and finished a work we started with Satoshi KOIKE two years ago.

This work whose aimed to establish a generalization, to the non-isolated singularities, of results contained in a paper we have published some years ago in the journal "Topology".

This last mentioned work was concerning the relations between the Kuo condition and an inequality of Thom's type, and it's consequences to the theory of Topological sufficiency and determinacy of mappings.

The generalization to the non-isolated singularities, we have accomplished, is not strait forward, we had to face many other problems, like the corresponding notions of the so called "jet extensions" and their realisations (depending on the geometry of their singularities). This has to be clarified before any attempt of extension to this case of the previous results.

The next challenge was to find in each case: topological-sufficiency, variety-sufficiency, etc. . . , the right condition which characterize such properties.

We have overcome all this problems, and end up with, in my opinion, an instructive and rich theory.

This paper, of about forty pages, will be submitted to publication.

**Impressions and thoughts on the present state of science in Japan in your field**

My field singularity theory and its applications, my research are in singularity and topology of functions, maps, spaces; and stratification theory.

My overall impression is that the research in both theory and the applications of singularity theory, is as good if not better than similar research at major Universities in Europe or the US.

Collegial ties among Japanese researchers are remarkably broad and strong, generously shared by my Japanese colleagues. There is an extraordinary level of cooperation in the research endeavor in Japan generally, and this is further promoted by governmental support in the form of grants and fellowships.

**(c) Comments or suggestions to HUTE, especially concerning this program**

The Fellowship program is extremely useful and fruitful. I am impressed by the rapidity of the positive reaction of the HUTE administration to my fellowship application and the help they provided to me. All my gratitude goes to them.

The greater cultural awareness that the Fellowship Program promotes is valuable and cannot be understated. The intellectual intensity, rigor, enthusiasm, and work ethic of Japanese mathematicians and in general Japanese scientists, are extraordinary, and it is infectious to the visitor. The benefits to the foreign scholar of a personal visit greatly outweigh those that might be achieved using simply long-distance written, voice or electronic communication.

**(d) Other comments**

**Thank you very much for your support.**